Phys 402
Fall 2022
Homework 2
Due Wednesday, September 14, 2022 @ 10 AM as a PDF file upload to ELMS

1. Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 5.1 (The quantum 2-body problem, i.e. transforming the hydrogen atom to the center of mass and relative coordinates) [*Two-particle H-atom reduced to two 1-particle problems. Note that you are solving Eq. (5.7) where particle 1 is the proton and particle 2 is the electron.*] [*Hint for part (a): Let* $\vec{R} = (X, Y, Z)$ *and* $\vec{r} = (x, y, z)$. *Then the x-component of the gradient operator acting on particle 1 is:* $(\nabla_1)_x = \frac{\partial}{\partial x_1} = \frac{\partial X}{\partial x_1}\frac{\partial}{\partial X} + \frac{\partial x}{\partial x_1}\frac{\partial}{\partial x} = \left(\frac{m_1}{m_1+m_2}\right)\frac{\partial}{\partial X} + (1)\frac{\partial}{\partial x} = \frac{\mu}{m_2}(\nabla_R)_x + (\nabla_r)_x$, *etc.*]

2. Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 4.28 (Classical interpretation of electron spin angular momentum)

3. Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 7.1 (delta function perturbation in the 1D infinite square well)

Additional part (c): Under what condition is perturbation theory applicable to this problem? Obviously α should be small enough, but small compared to what? Write a quantitative inequality that α should satisfy for the perturbation theory to be valid.

4. Second-Order Non-Degenerate Perturbation Theory: Starting with Griffiths Eq. [7.8] for the 2nd-order terms in the perturbation expansion, use the "sum buster" technique, and what is already known about ψ_n^1 from first-order perturbation theory, to derive the expression for the second order correction to the energy, Griffiths [7.15],

$$E_n^2 = \sum_{k \neq n} \frac{\left| \int \psi_k^{0*}\, \mathrm{H}'\, \psi_n^0 \, d^3r \right|^2}{E_n^0 - E_k^0}$$

5. Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 7.5 (a) **only** (2nd-order perturbation theory for problem 3 above]

6. What follows is a way to "derive" quantum mechanics starting from classical mechanics, following the logic of P. A. M. Dirac. We consider first the **Poisson Bracket** (PB) from <u>classical</u> physics, which is defined as follows. Consider two dynamical functions of the generalized coordinates and conjugate momenta of a single particle: $g(\vec{q}, \vec{p})$ and $h(\vec{q}, \vec{p})$, where $\vec{q} = (q_1, q_2, q_3)$ and $\vec{p} = (p_1, p_2, p_3)$ is the position and momentum of a single particle in three dimensions. Examples of g and h include components of angular momentum, a component of linear momentum, mechanical energy,

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linear kinetic energy, rotational kinetic energy, etc. Define the PB of g, h as $[g, h] \equiv \sum_{i=1}^{n} \left\{ \frac{\partial g}{\partial q_i} \frac{\partial h}{\partial p_i} - \frac{\partial g}{\partial p_i} \frac{\partial h}{\partial q_i} \right\}$, where n is the dimensionality of the system. (Note that $q_1 = x$, $p_1 = p_x$, etc.)

a) Prove from the definition of the PB that $[g, h] = -[h, g]$.

b) Show that for a single particle in 3 dimensions $[q_j, q_k] = 0$, $[p_j, p_k] = 0$, and most interestingly $[q_j, p_k] = \delta_{kj}$, where $j, k \in \{1, 2, 3\}$ and $n = 3$, and δ_{kj} is the Kronecker delta. {*Hint*: use the fact that $\vec{q}, \vec{p}$ are the independent variables that describe the state of the particle, i.e $\frac{\partial q_j}{\partial p_k} = 0$, etc.}

If the PB of two dynamical quantities vanishes, then the quantities are said to **commute**. If the PB of two dynamical quantities is equal to 1, then the quantities are said to be **canonically conjugate**.

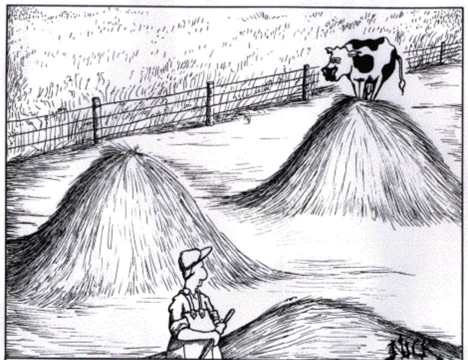
Starting with this, Dirac noted that the essential new ingredient of quantum mechanics (QM) is that certain observables [represented by operators $(\hat{u}, \hat{v})$] give different answers depending on the order in which the observables operate on a QM wavefunction, or in other words $\hat{u}\hat{v} \neq \hat{v}\hat{u}$. To account for this, Dirac re-defined the PB for the quantum case as follows: $i\hbar[u, v] \equiv \hat{u}\hat{v} - \hat{v}\hat{u}$. This leads to the following statements of the "fundamental quantum conditions" for the quantum position and momentum operators: $\hat{q}_r\hat{q}_s - \hat{q}_s\hat{q}_r = 0$, $\hat{p}_r\hat{p}_s - \hat{p}_s\hat{p}_r = 0$, and $\hat{q}_r\hat{p}_s - \hat{p}_s\hat{q}_r = i\hbar\delta_{rs}$. From this statement, one can derive many important results in quantum mechanics, as outlined in Dirac's book *Principles of Quantum Mechanics*.

c) Given the definitions and Dirac's argument above, find the quantum mechanical commutation relations corresponding to these pairs of classical dynamical functions: Two Cartesian components of the angular momentum vector in 3-dimensions, namely L_x and L_y (see Eq. (4.96)); L_z and x; L_z and p_x. Check your results against the quantum commutators given in Eqs. (4.99) and Eqs. (4.122) of Griffiths.

EXTRA CREDIT

2. The Stark Effect in Hydrogen. In 1913 Stark observed a splitting of the Hydrogen Balmer series lines by applying an electric field $E = 100{,}000$ V/cm.

   a) Write down the perturbing Hamiltonian $\mathcal{H}'$ for the electron. Neglect spin in this problem.
   b) Calculate the change in energy of the ground state of Hydrogen to first order.
   c) Consider the $n = 2$ states of Hydrogen. Find the new energies to first order.
   d) Calculate the new $n = 2$ eigenfunctions.
   e) Roughly calculate the resulting splitting of the Balmer series H$_\alpha$ line in an electric field of 100,000 V/cm.



PHYSICS AT THE FARM: DISCOVERY OF THE MUON.